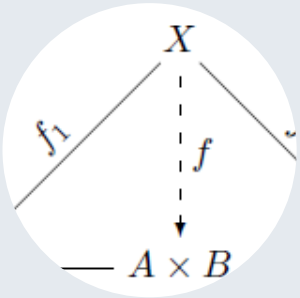

Defining a product

Grad. Algebra I

MATH B503

Course info

- Tuesday (resp. Thursday)
- 12:55–2:15p (resp. 8:25-9:45a)
- Park Science 328

Instructor

- Professor John Bergdall
- Park Science 334
- jbergdall@brynmawr.edu
- x5356

- Office Hrs:
  Mo 12:00–1:00p
  Wed 9:00-10:00a
  Thu 11:30a-12:30p
  Fri 4:00–5:00p*
  (*most weeks)

Problem session

- Thursday
- 12:55–2:15p
- Park 328

Essential questions

The three essential questions MATH B503 aims to explore are:

- How are mathematical structures organized?
- How are definitions in mathematics given and used?
- How are mathematical objects built from one another?

Longer overview

This is a 1st semester course in algebra at the graduate level. Just as in your introductory algebra courses, we will study groups, rings, and fields. The difference here is viewpoint. We place a higher emphasis on definitions and structural relationships as opposed to quantitative facts. The mechanism used to manage relationships is the language of *category theory*. To quote directly from our text (Section I.3):

> The language of categories is affectionately known as *abstract nonsense*, so named by Norman Steenrod. This term is essentially accurate and not necessarily derogatory: categories refer to *nonsense* in the sense that they are all about the 'structure', and not about the 'meaning', of what they represent. The emphasis is less on how you run into a specific set you are looking at and more on how that set may sit in relationship with all other sets. Worse (or better) still, the emphasis is less on studying sets, and functions between sets, than on studying 'things, and things that go from things to things'...

We hope you get a sense of our text from that final sentence, but let us dwell on it for a moment. In the 17th century, in western Europe, it was still a great problem to determine areas and arc lengths of smooth figures. Calculus was invented to solve these, but it's worth recognizing that its utility depends on our ability to both describe and calculate *simultaneously*. This was made possible by using algebraic variables (x, y, and so on) emphasized by Descartes, who adapted the tradition of algebra that flowed out of India and Persia in the centuries prior. Overnight, difficult problems that were translated into questions of functions of variables, which *a priori* are just "things," became easier to solve based on simple maneuvers (e.g. the chain rule). The systematic adoption of variables allowed mathematicians to gain crucial practice and experience by working with general equations. The power of this shift is felt today, where modern mathematics is as comfortable with "things" as it is with numbers (c.f. probability spaces or moduli spaces).

In this course we will study the most basic problems in algebra: questions of construction and de-construction of "things." In one sense, these problems are the simplest to state, the easiest to solve, and they use the most general techniques. We study them because they have proven to be the most useful for general mathematicians to understand and they serve as crucial training for more advanced topics.

This course will serve you, perhaps only in the future, with models for how mathematical progress is made through isolating key concepts. We will practice making and using definitions in a 21st century style. You will, of course, also learn some awesome techniques and theorems in algebra. And finally, you're going to have fun!

Learning goals

In MATH B503, you will:

- Develop advanced skills in reading, writing, and discussing mathematics.
- Learn how mathematical objects are defined by their relationships with others.
- Practice categorical thinking and recognizing *content* versus *formality* in math.
- Study the main structure results about finite groups (e.g. Jordan–Hölder), modules (over principal ideal domains, say), endomorphisms of vector spaces (e.g. canonical forms), and roots of polynomials (e.g. Galois theory).

FAQs

How should we address you?

I use he/him/his pronouns. If you need to name me, use Professor Bergdall or John (not Dr.). If you write me an email, include an appropriate greeting (Dear/Hello...) and closing (Sincerely/Thank you...).

What if I forgot xxx?

We will spend the first month somehow reviewing, so you should not worry deeply about what you've forgotten. But, we will not review your undergraduate course literally. As we recall basic objects with a fresh air, you will want to check-in to see if you need further reminders. We are always happy to sit and talk algebra!

What if I learned xxx already?

We encourage you to look at everything with fresh eyes, even if you've seen it before. In a class with diverse algebra backgrounds, there is going to be some overlap with your past courses. You should take the chance, having seen things once, to look from new perspectives. If all else fails, remember: anything worth proving once is worth proving twice.

Will you repeat that? Can you say it more directly?

Yes, gladly! Part of mathematical discourse for me is EXCITEMENT! Please *selfishly* ask me to repeat or re-explain anything you have not grok'd.

Prerequisites

To be prepared for this course, you should have been successful in a course where you had (i) significant experiences reading and writing proofs, and (ii) exposure to basic structural results on, and *examples* of, groups, rings, and fields.

We emphasize "examples" because we will not focus in-depth on any one group, ring, or field. Instead, we will study relationships among them. Doing well in either year-long algebra sequence in the BiCo is sufficient preparation. If you are unsure, come speak with us.

Material

Text

Allufi, P., *Algebra: Chapter 0*. 1st edition. American Mathematical Society.

Our book has been chosen for the overall clarity of its writing and the modern viewpoint. The author intends, and we expect, for you to read nearly every page of the text that we cover. (I promise, it's enjoyable!)

The author maintains a list of errors at

https://www.math.fsu.edu/ aluffi/algebraerrata.2016/Errata.html

Other texts you may want to consult are discussed later.

Discussion forum

Piazza is a question and answer site. Our class-specific forum is located at

https://piazza.com/brynmawr/fall2019/mathb503/home

though you will need the URL

https://piazza.com/brynmawr/fall2019/mathb503

in order to sign up. We have dedicated a section below to discussing Piazza.

Accessibility

Bryn Mawr College is committed to providing equal access to students with a documented disability. Students needing academic accommodations for a disability must first register with Access Services. Students can call 610-526-7516 to make an appointment with the Access Services Director, Deb Alder, or email her the address dalder@brynmawr.edu to begin this confidential process. Once registered, students should schedule an appointment with the professor as early in the semester as possible to share the verification form and make appropriate arrangements. Please note that accommodations are not retroactive and require advance notice to implement. More information can be obtained at the Access Services website whose URL is http://www.brynmawr.edu/access-services/

Any student who has a disability-related need to tape record this class first must speak with the Access Services Director and to me, the instructor. Class members need to be aware that this class may be recorded.

Academic Integrity

The Bryn Mawr College Honor Code is in effect for all students enrolled in this course. We also provide some specific guidelines for different assignments in *this* course below.

Other recommended texts

The following texts should be on reserve in Collier Library.

- Dummit, D. and Foote, R. *Abstract Algebra*. John Wiley & Sons. ISBN: 0-471-43334-9.
- Lang, S. *Algebra*. Graduate Texts in mathematics, 211. Springer-Verlag. ISBN: 0-387-95385-X.
- Hungerford, T. *Algebra*. Graduate Texts in Mathematics, 73. Springer-Verlag. ISBN: 0-387-90518-9
- Isaacs, I. M. *Algebra*. Graduate Studies in Mathematics, 100. American Mathematical Society. ISBN: 0-8218-4799-2.
- Jacobson, N. *Basic Algebra I, II*. (These are printed these days by Dover).

You should not need these texts to do well in our course. They may be useful later in your life. Dummit and Foote is rife with examples, hence useful for first exposure to a topic. Lang is invaluable both as a desk reference and as a tofu press. The others are common choices for learning and reference. Every so often we also open the Bourbaki *Algébre* chapters.

Grade overview

> ⚠ *You will receive an above merit grade (either S or at least a 2.0) if and only if you pass every assignment.*

Grading in a 500-level course depends on which capacity you are enrolled as.

- Students taking the course for only graduate credit (this applies to all the Ph.D. students) receive a grade of S (satisfactory) or NS (not satisfactory). You will also receive narrative feedback after the final exam.
- Undergraduate students taking the course for elective credit receive a grade on the a.b scale, as usual.
- Students receiving both elective and graduate credit receive both kinds of grades. Lucky you!

For each assignment, we will determine a score based on the quality of the work. These scores will be compiled into a course grade according to the following breakdown:

5%	Daily course contribution
15%	Problem sets
15%	Reading assignments
10%	Midterm conversation
20%	Final conversation
35%	Written final

The final grade lines will be *at least* as generous as: >90% earns a 4.0, >80% earns a 3.7, >70% earns a 3.3, and so on.

Late work policy

For problem sets and reading assignments:

- If you turn in a homework assignment late, we will deduct 25% per week, rounded up. (Late at all = one week.)
- If you "ask" 12 hours ahead of the due date, you may turn in a homework assignment up to 48 hours late with a 10% deduction, instead. Ask is in scare quotes because you need to only email us announcing your intention. If you tell me in person, please also email me (as soon as possible).
- You get two "late passes" to use during the semester. You may once turn in any homework one week late without penalty, and you may once turn in any homework 48 hours late without penalty.

For conversations and the written final:

- Both the conversations and written final have a significant window in which they may be completed. So, we only anticipate allowing extensions or make-ups in the presence of debilitating and unavoidable obstructions (overwhelming concerns of wellness, emergency travel, military duty, etc.). Everyday commitments (like other tests) or opportunities (like leisure travel) are not a valid excuse. In the event you do need a make-up or extension, you should interface with us as soon as possible. You should also include your dean or an adviser in the conversation, if appropriate.

Details on graded components

Daily course contribution

To pass your daily course contribution you must:
⚠ Make at least one contribution to an in-class discussion and one contribution to a Piazza discussion by October 4.

You are expected to make contributions to the course. Passing will depend on making an early definite contribution. A contribution consists of a detailed question, answer, or comment. "What does that say?" is not a detailed question; "Can you re-explain the 3rd step of that proof again, more conceptually?" is. We set an early deadline to set the tone for a collaborative environment. If you have concerns about meeting our expectations, please interface with us.

For attendance, *you are expected to be at lecture and the problem session, on time, on a daily basis.* Regularly missing class or arriving late, even by five minutes, is disruptive. If you miss multiple classes in a row, or too many over a short period, you will have an extremely difficult time keeping up. We will track attendance and account for it as follows:

- You will begin the semester with 8 points.
- We will deduct 1 point for missing a lecture or problem session and a $1/2$ point for being late ("after announcements").
- If we are late by the clock, everyone will earn a $3/4$ point. (You can have more than 8 points.)

At the end of the semester, your score will be calculated out of 5 points. The 3 "free" points account for typical absences.

Students in the Tri-Co have met our expectations when they are clear. This policy clarifies our expectations; it is not meant to punish. You may encounter a situation, or situations, during the semester that stop you from regularly attending lecture on time. If so, it is your responsibility to interface with us as soon as possible so that we have the chance to adapt our expectations to your changing situation. You are invited to utilize your dean or other adviser to communicate with us.

Problem sets and reading assignments

To pass the problem set component you must:
⚠ Turn in earnest efforts at enough problems to meet the minimum number of ⋆'s required for each problem set.
To pass the weekly reading component you must:
⚠ Complete every reading guide.

Problem sets and reading assignments are opportunities to practice the material discussed in lecture and develop skill in reading mathematics. They will test your understanding of basic concepts, amplify the purpose or utility of viewpoints, and reveal areas you need to work harder to understand. Both will be due on Tuesdays at 6:00p.

On problem sets there will be a mix of compulsory and optional questions. Each will be given a ⋆ rating and your assignment will require a minimum number of ⋆'s. The focus of the problem sessions will be to discuss solution strategies. Our course grade lines are generous, so try some harder optional problems. They will often interface with other fields, like topology and analysis.

This class is about exposures to ideas. Not doing a problem set will result in not being exposed to an idea. So, every problem set must be turned in to pass the class, and you may not turn in a problem set until it meets the minimum number of ⋆'s. You may turn in *more* than the minimum number of ⋆'s. Every problem will be graded on a 1-5 scale. Your numerical score will be the percentage of points you earn among all the problems you turn in. If you turn in more ⋆'s than required, you will not receive extra credit, but you will receive extra feedback! If you think you're struggling in the course, we highly recommend trying more than the minimum number of ⋆'s or *at least* brainstorming strategies to do all the problems. You *are* capable of earning strong scores, but it may take a minute and you may need help re-aligning your viewpoint of the material.

One of the learning goals in this course is to develop your skill in reading mathematics. Our textbook has been chosen for its remarkably high-quality writing. (How does this meager syllabus compare?) We want to take advantage of this and offer you the chance to closely read material on your own. As a class, this also allows us to move at a quicker pace.

Some reading will be assigned ahead of lecture. You are responsible for reading not covered during lecture. We will provide a brief guide each week to help you organize that information. A specific portion of the text will be emphasized on the guide, and you will be asked to read it closely and answer some related questions.

⛔ *Honor code guidelines*:

When working on homework, you can discuss with anyone in the class, make liberal appropriate use of the Piazza forum, consult the text and your notes, or do independent reading beyond the text (including taking advantage of the internet!). However, the work you turn in needs to *written independently and reflect your understanding of the material*. Turning in work to which the italicized text would be a violation of the honor code. If you do significantly consult other resources or people while preparing your solutions, you should acknowledge that. (This will help us craft future problem sets and understand better which texts illuminate which topics the most.)

Conversations (midterm and final)

To pass the midterm conversation you must:
⚠ Exhibit the ability to accurately explain *some* of the core concepts from the first unit of the course.

To pass the final conversation you must:
⚠ Complete the final conversation.

Having a quality 30 minute conversation with a fellow researcher is worth days of reading their writings. Visiting a collaborator for a week to do intense research will pay dividends roughly on the order of spending a year collaborating over email.

So, one of the learning goals in this course is to develop your skill in discussing mathematics. Twice during the semester you will be invited to our office to have a brief conversation about topics from the course. The dates and lengths for these conversations will be:

- Midterm conversation: October 9-11 (30 minutes each).
- Final conversation: December 9-12 (60 minutes each).

The topics for the midterm conversation will be from the first unit of the course (Chapters I, II, and III of the text). You will not know the questions ahead of time, but everyone will get the same basic starting ones. Possible formats for questions are:

- Would you please define xxx?
- Would you please state and prove Theorem xxx? Explain all the hypotheses you need.
- Would you please provide an example of xxx? Explain.

To prepare, focus on the large "structures" we discuss and how they are instantiated in examples. We will be discussing the most important objects for the rest of the class. Make sure you know their definitions and properties!

Though the conversations will begin the same for each student, it is impossible for them to remain that way for long. As our conversation progresses, we may ask you to expand on, or clarify, a point. If you struggle, that is okay. We will provide hints or examples to help. If you are doing well, we may move onto new questions. Or, since this is a conversation *between* two people, *you* may choose to add something deeper if you feel you are especially expert on a topic.

The midterm conversation is diagnostic for being in position to succeed in the rest of the course, and it will prepare you for the final conversation. The main difference at the end of the course will be that our conversation will be twice as long and the topics will be randomly chosen (so possibly different) for each student. You will also likely have to discuss proofs of Big Theorems (there are none in the first unit). When you come to our office, you will find out your topics and then you will have one hour to collect your thoughts and prepare.

For either conversation, you will be recorded so that we may listen back. This is *not* to hunt for mistakes, but to hunt for *knowledge*. We want to capture your knowledge *as much as possible*. (If you like, we can share the recording with you.)

Passing the final conversation only requires completing it. Passing the midterm conversation requires slightly more. If, on the day of our conversation, you have extraordinary difficulty explaining the core concepts, then you will be asked to repeat the assessment during the week October 28-November 1.

⛔ *Honor code guidelines*:

For either conversation, you may prepare however you like. Consult any and everything. For the final conversation, once you know your topics, you may subsequently spend your free hour writing *fresh* notes to use. (Copying and pasting is not writing.) For either conversation, you should avoid discussing with a peer until after the conversation period has ended.

Written final exam

To pass the take-home exams you must:
⚠ Make progress toward a solution on 1/2 of the problems.

During the finals period there will be a take-home, open note, and open book exam. The exam will be handed out in envelopes following the last problem session. (If you are afraid you will lose it, we can hold onto your exam until you know when you want to start it.) It will be due at the end of finals period (December 20 at 12:30p). You should spend no more than 72 continuous hours working on the exam, including writing up your solutions. While preparing for the exam, you may consult any resources. On the exam itself you will be allowed your notes and text.

(⛔) *Honor code guidelines*:

You should not open the exam until you are ready to begin your 72-hour window. During the window, use only your notes and book. Before you begin your exam, avoid discussing the exam with anyone you are aware has started. After taking the exam, avoid discussing the exam until the finals period is over. If you use a computer to type your solutions, be diligent that you do not consult extra resources. Of course, you can listen to music, use your computer as a clock, etc. If you decide to watch a movie, avoid "It's My Turn" or any other film that may inadvertently contain the proof of a problem ☺.

Piazza

The question and answer discussion will keep flowing on Piazza once we leave the classroom. Note: Piazza allows for TEX to be used, so we can actually read the mathematics. The Piazza environment will serve us in three ways:

- It provides an opportunity to practice developing skills in reading, writing, and discussing mathematics.
- It replaces email as the main forum where questions about concepts and material can be asked and answered.
- Students not on the Bryn Mawr campus can ask questions on Piazza if they cannot make it to office hours.

The first thing you may wonder is: What goes on the Piazza forum? The answer: Basically everything.

- If you ask a question over email about material, there is a 95% chance we'll ask you to put your question on Piazza.
- If you ask a question over email about logistics, there is 75% chance we'll ask you to put your question on Piazza.
- If you ask a question in office hours, there is a non-zero chance we'll ask you to put your question on Piazza after you leave. We may also ask you to sketch an answer if you've come up with one.
- During lecture, we will sometimes make a small symbol (like ⓟ) to indicate there are details to be filled in (or concepts to clarify) but you should do it at home. These little details are *perfect* for a Piazza discussion!

Before getting into ground rules, we should point out:

> ⚠ *The questions and answers wiki-editable. The entire class is responsible for their accuracy and clarity.*

In practice, this means that if comments reveal a question or answer could be improved, then everyone is responsible for making sure the appropriate edits are made. That may mean you have to read the discussion to make sure you understand. But won't that involve learning some mathematics? Perfect!

There are a few ground rules for using Piazza. First, let's deal with asking questions:

- You may not literally ask for solutions.
- Your question should include precise references to book page numbers, exercise numbers, etc. if appropriate.
- Your question should include what you have tried, what you want to try, and what is stopping you from trying. ("I think I need to use G is abelian, but I don't see how that interacts with Theorem xxx...") In our experience, even the *act* of trying to formulate a question will reveal the next steps.
- You should address your peers respectfully and thank them for their help.
- You should isolate out *one question or topic* per post. If you have more than question, please create subsequent posts.

For answering questions:

- Directly answer your peers' question without going deeper into solutions (if applicable).
- If you feel uncomfortable answering a question, leave a *comment* suggesting how the question could be improved.
- Answer questions respectfully. Put downs (even implicit ones like calling a question "trivial") will not be tolerated.

You can also comment on questions and answers. Comments are great for clarifications and suggesting improvements.

- If you need a clarification or have a suggestion, write kindly. Challenge ideas, not people.

Assignment deadlines

Major assignment dates

Assignment	Start date	End date
Midterm conversation	10/09	10/11 (12p)
Final conversation	12/09	12/12 (5p)
Written final	12/12 (2:15p)	12/20 (12:30p)

Weekly assignment schedule

Assignment	Out day	In day
Problem sets	Tuesday (12:55p)	Tuesday (6p)
Reading guides	Tuesday (12:55p)	Tuesday (6p)

Beginning class schedule (minimal and tentative)

UNIT 1: Basic categories of algebra (8 lectures)

Date	Leading Q.	Topics
09/03	Who are we and what do we want?	Syllabus, categories and universal properties [A, I.3-5]
09/05	How do we see groups through the lens of category theory?	The category Grp and examples [A, II.1-5]
09/10	How do we construct new rings?	Subgroups and quotient groups [A, II.6-7]
09/12	What else should we remember about groups?	Cosets and group actions [A, II.8-9, IV.1]
09/17	What else is a good example of a category?	Rings and universal constructions [A, III.1-2]
09/19	How do we construct new rings?	Quotients, maximal and prime ideals [A, III.3-4]
09/24	But...wait...you take quotients of rings by not rings?	Modules, noetherian-ness, algebras [A, III.5-6]
09/26	What is the calculus of modules?	Complexes and homology [A, III.7]

First day check-in

Some personal details

Preferred name:

Preferred pronouns:

Logistics

Are you registered for this course, definitely planning to register, or shopping?

☐ Registered ☐ Planning to register ☐ Shopping

My BiCo username is: ☐ @brynmawr.edu ☐ @haverford.edu

Briefly describe your experience with algebra:

Goals

Describe one goal you have for *this* course:

Describe one goal you have for this *semester* (regardless of MATH B503):

Class formation

Course discussion will occur in-class and online for us. Briefly describe one norm you would like the class to observe during discussion. (These will be compiled and posted to moodle *anonymously*.)

In the remaining space, if you are comfortable, please indicate anything else you think it is important for us as your professor to know. (If you are not comfortable, please email us or come see us in office hours.)

Classroom norms

- If possible, I'd appreciate any and all feedback. I find that it's very important to have an outside point of view to improve.
- Respect all questions.
- Try not to assume everyone knows/remembers everything you do. Good to bring up old information, just don't take it for granted. (Something I used to do, so if you catch me doing it, glare at me or something.)
- Don't use language that minimizes problems/proofs such as "easy," "pretty," "cute," etc.
- Balance comments with a kind word. Relate openly with someone else on a confusing point. Strength in numbers!
- A discussion should include more than one student's voice (or a particular subset of students' voices).