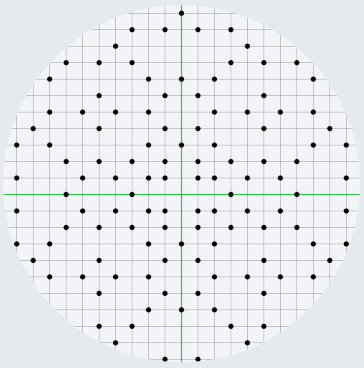

Gaussian primes

Algebra II

MATH 304

Course info

Tuesday & Thursday

8:25-9:45a

Park Science 336

Instructor

Professor John Bergdall

Park Science 334

jbergdall@brynmawr.edu

x5356

Office Hrs:
M 2:30-3:30p
Tu 11:30a-12:30p
Th 1:30-2:30p

TA info

Daniel White

Park Science 436

dfwhite@brynmawr.edu

TA sessions

Session 1 (90 minutes)

Th 12:00-1:30p

Park 328

Session 2 (90 minutes)

F 2:30-4:00p

Park 328

Overview

This is a second semester course in abstract algebra, focusing on the theory of rings and fields. In this course you will explore some big questions in mathematics: What does it mean to solve equations? In what ways are algebraic structures related? How do we (humans) organize and simplify mathematical information? How do the techniques I've been previously trained in translate to more general situations?

The chief aim of this course is detailing Galois theory. The history of Galois theory is astounding (politics! love! duels! death!) but our main interest lies in the singular nature of Galois theory to give a glimpse into a thoroughly modern from of mathematics epitomized, as you will see, by the way in which Galois theory connects two completely disparate areas of mathematics. Namely, the theory of symmetries as emphasized by group theory is joined to the algebraic theory of equations in a single fluid correspondence.

Material

Required texts

Judson, Thomas W. *Abstract algebra, theory and applications*. 2018 Edition.

Available for free in electronic form.

Recommended texts

Fraleigh, John B. *A first course in abstract algebra*. 7th Edition. Pearson. 2002.
Herstein, I.N. *Topics in algebra*. 2nd Edition. Wiley & Sons. 1975.
Dummit, David S. and Foote, Richard M. *Abstract algebra*. Wiley. 2003.
Artin, M. *Algebra*. Prentice Hall. 1991.

Available from the reserve shelf in Collier Library (BMC).

Learning goals

- Learn how to logically reason across correspondences in mathematics.
- Understand the utility and practice of passing between abstract theory and calculations.
- Learn the how and the why of reframing concrete questions into abstract frameworks.
- Practice writing, editing, and revising mathematics in teams.

Grade breakdown

10%	Daily course contributions
10%	Problem sets
15%	Mastery and skill assessments
25%	Final exam
40%	Lecture publishing

The default grade lines will be: >93% earns a 4.0, >90% earns a 3.7, >87% earns a 3.3, >83% earns a 3.0, and so on. We reserve the right to make the gradelines *more* generous.

Office hours and appointments

Office hours are listed on the left-hand column. Appointments will be available, but initially restricted to certain periods for ease of scheduling. Scheduling available through moodle. Requests must arrive *12 hours* in advance of the appointment.

FAQs

Do we need to know how to use LaTeX?

Yes! You'll be producing lecture notes for you and your peers, and these must be typed. You should come see me as soon as possible if you are not familiar with LaTeX.

Do I need group theory for this course?

Yes, to a degree. You will need to eventually recall the structure of finite abelian groups (though we will rederive that) and you will want to remember what a normal subgroup. But do not worry! You have at least 6-8 weeks until it comes up!

What is your favorite result?

Zorn's lemma. It unlocks all the power you lose by restricting your mind to finitely many dimensions!

Which extra book is the best?

This is a tough call. For pure enjoyment of reading, Herstein's is the best. But fair warning: Herstein uses very old notation that no modern mathematician would! Fraleigh is a little lighter on the touch and goes thoroughly through examples. Dummit & Foote should largely be read selectively as a reference. My recommendation is to get to know these books in your library. I also have copies in my office you are invited to borrow for short periods.

Daily course contribution

10% of your grade is based on your daily course contribution. We expect you to completely earn these marks, which you can regardless of your *vocal* participation in the classroom. The key is we need you actively engaging in the material. If we feel that you are not, you may be approached and we will have a discussion in case there has been a misunderstanding.

Part of our expectations for your contribution, though, is that *you are expected to be at lecture, on time, on a daily basis.* Regularly missing class or arriving late, even by 5 minutes, is extremely disruptive and we will take this into account through the daily course contributions grade.

- Each missed lecture results in a 7% reduction of that component.
- Arriving late, determined by whether announcements for the day have finished, will count as 1/4 of a missed lecture.
- Deductions will not begin until after your 3rd missed lecture.
- If we are late, everyone gets a free 1/2 lecture credit.

Late Work and Make-up Policy

- You may turn in two problem sets up to a *72 hours* late without penalty or permission. Otherwise, it is a 10% deduction per 72 hours, rounded up. (Late at all earns a 10% deduction, late by 80 hours earns a 20% deduction, etc.)
- An extension on a mastery assessment will only be allowed, with our approval, for students who make a *request* at least *24 hours before the quiz is due*.
- Deadlines for components of lecture publication will vary, but they are mandatory once determined. Late or missed work will count as zero.

Accessibility

Bryn Mawr College is committed to providing equal access to students with a documented disability. Students needing academic accommodations for a disability must first register with Access Services. Students can call 610-526-7516 to make an appointment with the Access Services Director, Deb Alder, or email her the address dalder@brynmawr.edu to begin this confidential process. Once registered, students should schedule an appointment with the professor as early in the semester as possible to share the verification form and make appropriate arrangements. Please note that accommodations are not retroactive and require advance notice to implement. More information can be obtained at the Access Services website whose URL is http://www.brynmawr.edu/access-services/

Any student who has a disability-related need to tape record this class first must speak with the Access Services Director and to me, the instructor. Class members need to be aware that this class may be recorded.

Academic Integrity

The Bryn Mawr College Honor Code is in effect for all students enrolled in this course. We also provide some specific guidelines for *this* course.

We expect you to consult multiple sources (other textbooks, peers, office hours, etc.) in the process of the learning the material. However, any written work you submit should be *in your own words* and *reflect your own understanding*. The easiest way to achieve this, and an excellent way to make sure you understand what you are writing, is to write up each piece of your final work alone and away from any notes you've gathered. Remember, whether you are typing or handwriting an assignment, there is no need to create the "final product" all at once — you can write up a final version of a problem before moving onto solving the next one.

Should you find it impossible to separate your final work from someone else's work, or something you read in an extra resource, we expect you to indicate that in some way. Helpful phrases include: "This argument is based on..." or "We will argue similarly to the argument in..." Again, your end product should be *in your own words* and *reflect your own understanding* and so including this information is simply acknowledging where you received help from.

Video lectures and "just in time" exercises

Lectures in this course will move quickly. Being 80 minutes long, they will often contain multiple new definitions or concepts.

- You will be assigned a pre-lecture video to watch and a short exercise to complete ("just in time").
- Optionally, turn in the exercise *at the start of lecture* and we will scan them and leave comments if possible.

Typically, the video will summarize the upcoming lecture and then go deeper into one specific concept. The exercise meant to check your understanding *before* lecture. Videos will be available through the moodle, following the preceding lecture.

Problem Sets

Exercises will be assigned throughout the course.

- Exercises assigned in class, written on the board and indicated with a $\star$, are for practice and you will self-assess your solutions. (But check below re: lecture publishing!)
- Organized problem sets (4 problems) will be graded on a weekly basis.

Problem sets will be handed out on Tuesday morning and will due by Tuesday *evening at 5:00 PM*.

- If you are writing lecture notes, then you are excused from the problem set. (You are responsible for the results, though.)

Mastery and skill assessments

The course schedule is broken into 4 large topics. For each topic you will be given a mastery and skill assessment (a long-ish quiz). The format of these will be *take-home*, *closed notes*, and *timed* (90 minutes). The questions will significantly range in difficulty. These will count as 15% of your grade. We will only consider your three best efforts. Upcoming skill assessments will be announced at least a week ahead of time.

Final exam

There will be a self-scheduled final exam. You will be allowed one textbook of your choice (you must provide me with a copy if it is not on the list above) and our course lecture notes. The final exam will be worth 25% of your final score.

Lecture publishing: purpose

Lecture publishing is the most exciting, and complicated, portion of this course! At the 3xx-level of mathematics, students discover there are many paths to explaining the material, each with their own benefits and drawbacks. Throughout the semester you and your peers will regularly produce lecture notes in a small group (3-4 people per group). Ideally, you will be in charge of 4 lectures, but that could change depending on course enrollment. Your main goal will be to make available a polished public draft of lectures notes, including solutions to video and in-class $\star$ exercises, 5-7 days after your assigned lecture. In the weeks that follow, you will revise your notes based on peer review.

Lecture publishing: group roles

There are a number of roles we would like your group to assign in order to divide up the work. Each group is free to determine how to structure themselves. For instance, a group of 4 might want a notetaker, a writer, and two editors. Another group may want to evenly divide up the work each week (this is not so highly advised). A group heavy with notetakers may decide they will all take notes and then split up the rest of the work evenly. If you divide the roles per lecture, then the roles should rotate during the semester. Below I will describe the responsibilities of the roles for the *assigned lecture*.

Everyone

From the start, everyone is in charge of:

- Adding in the details left by $\star$ exercises in lecture and the pre-lecture video.

Your classmates are depending on you! It is best if you meet as a group very early to discuss strategies for these exercises! (Remember: you are not required to turn in a problem set this week.)

Notetaker(s)

The notetaker is responsible for:

- Creating a clean hand-written set of lecture notes.
- Determining and clarifying any gaps in the presentation during lecture.
- Disseminating hand-written material to the rest of the group.

On the middle point, it is possible an argument in class gets out of hand and a gap appears. You are responsible for (a) identifying that and (b) working to fix it. We invite you to use a (sound) recording device if it would help you catch everything.

Writer(s)/Typer(s)

The writer is responsible for:

- Transferring hand-written notes and pre-lecture video notes into type-written notes.
- Insert/include all necessary formatting such as labeling theorems or lemmas, referencing previous results, and so on.
- Developing a narrative for the lecture by writing introductory paragraphs and transition paragraphs between the topics.

Editor(s)

The editor is publicly responsible for the lecture notes. They will receive and distribute comments on typos, clarity of the notes, and so on. Specifically, they are responsible for:

- Turning the draft from the writer into a final production that is suitable for sharing with me.
- Arranging an editorial meeting between myself, themselves, and as many members of their group as possible.
- Disseminating and dividing revisions to their group.
- Posting the final public draft to moodle.
- Updating and uploading any subsequent revisions.

Lecture Publishing: Timeline

Now that we have explained the overview and the roles, let us go through the logistics.

- Notes for a given week need to be published to moodle by Tuesday *evening at 5:00 PM* (same time as problem are due).

So, note-taking on Thursday has a shorter turnaround than Tuesday. We will rotate the groups to even everything out.

Your group is responsible for creating a reasonable schedule you can stick to. The only restriction is:

- On Monday following your lecture, we need to have an editorial meeting to discuss your notes.
- You need to be prepared to incorporate significant changes between the editorial meeting and mid-day on Tuesday.

Below is a sample schedule you may wish to follow. It occurs over 4 days, so that notes from Thursday are ready by Sunday.

Day 1

- Everyone attends lecture and notetaker takes careful notes.
- Notes, including relevant pre-lecture video notes are photocopied/printed and disseiminated.
- Notetaker examines notes carefully to determine if there are gaps and what resources will help fill the gaps.
- All group members begin working together to fill in the $\star$ exercises.
- The editor schedules an editorial meeting for Monday.

Day 2

- The writer begins formally typing the notes, including a narrative structure and labels for theorems, definitions, etc.
- Partial solutions to the $\star$ exercises are added.
- Writer does small editing, enough to minimize typos and to ensure the logical structure is correct.
- Notetaker and editor work together to fill any gaps noticed in the lecture notes.

Day 3

- Editor works to create a polished draft of the notes, including checking carefully for language and typos.
- Editor also closes mathematical gaps in arguments, justification, or reasoning.
- Group determines final solutions to $\star$ exercises and those get inserted completely into the notes.

Day 4

- Entire group examines a post-edited copy and submits a .pdf to me before editorial meeting.
- Prepare for discussion at editorial meeting.

Editorial meeting (Monday)

- Myself, the editor, and as much of the group as possible, go through the notes.
- Comments and corrections are noted. Editor passes them to the group.
- Team works to revise/edit gaps.

Dissemination (Tuesday)

- Notes go through final review by editor. Remaining issues and gaps are indicated with footnotes.
- Notes are posted to moodle by *5:00 PM*.

Lecture publishing: review and revision

There will be three mechanisms for review of your notes.

- You may have posted your public notes without filling all the gaps. You will have a chance to revise them!
- Your peers may pass you information about issues with your notes, either privately or publicly. (I will create a public moodle forum for each class week for this purpose.)
- You will also receive anonymous peer reviews of portions of your notes.

Your peer reviewers will be assigned each Tuesday and they will have one week to complete their review. Once I receive all the reviews, I will compile them and pass them to you along with a generous date for your final revision.

Lecture Publishing: Grading

Let me break down how the lecture publishing process will be graded. Each lecture you are assigned will have 15 points associated with it. Most of the points are for meeting benchmarks.

1 pt	Schedule an editorial meeting.
2 pts	Arrive at editorial meeting with notes and group prepared.
5 pts	First public draft. All points awarded if notes posted on time with strong structure, minimal typos, issues from editorial meeting addressed. Points deducted if there are significant typos, writing issues, or major missing components. (Duly noted mathematical issues are okay still.)
2 pts	Incorporating review comments. Points awarded based on effort to incorporate or address reviewer comments.
5 pts	Final product, after peer review. Graded on: timely submission, correctness, completeness, organization, writing quality.

This is for each lecture. The overall lecture publishing component of your grade will then be broken down according to the following percentages.

70%	Lectures published. (Based on the above.)
20%	Final semester narrative reflection of participation in the group.
10%	Peer review of others' work. Graded on effort.

The middle point deserves clarification. In the final week of the course you will be asked to prepare a narrative describing how your group interacted and the ways in which you contributed to your group's success. I hope this can be read and attributed just for effort, but if significant issues of equity arise then I will consider this the place to address that.

Class schedule

Part I: Foundations of ring theory (7 lectures)

Part II: Modules and vector spaces (5 lectures)

Part III: Field extensions and Galois theory (8 lectures)

Part IV: Advanced linear algebra (6 lectures)

Detailed class schedule

Below is a reading schedule for the first two units of the course. Subsequent reading details will be given. Reading sources: [J] for Judson, [F] for Fraleigh, [DF] for Dummit & Foote, and [H] for Herstein. TBA refers to a class handout or scan from another text. (Note: The readings cover sections of textbooks, and we may not cover every detail of these sections *in class*.)

PART 1: Foundations of ring theory (7 lectures)

01/22	What are the aims of the course?	Syllabus & Introductions Rings and solving equations Morphisms of rings	[J] 16.1-3, [F] IV.18-19
01/24	How do we construct new rings?	Ideals and quotient rings Maximal and prime ideals	[J] 16.3-4, [F] V.26-27
01/29	What is special about polynomial rings?	Degrees of polynomials Euclidean division Introduction to Principal Ideal Domains (PIDs)	[J] 17.2, [F] IV.23
01/31	When are polynomials irreducible?	Gauß lemma Rational roots theorem Eisensteins' criterion	[J] 17.3, [F] IV.23
02/05	How does factorization work in general rings?	Unique Factorization Domainds (UFDs) PIDs Euclidean domains	[J] 18.2, [F] IX.45-46
02/07	How does this connect back to usual integers?	Gaussian integers Contents	[J] 18.2, [F] IX.47, [DF] p. 289-92

02/12	How do we construct new UFDs?	Field of fractions Gauß lemma (II)	[J] 18.1-2, [F] IX.45

PART 2: Modules and vector spaces (5 lectures)

02/14	How do rings behave as scalars?	Modules and vector spaces Free and quotient modules	[J] 20.1-2, [DF] 10.1-2, [H] 4.1
02/19	What if the scalars are invertible?	Vector spaces Linear independence and span Dimension and bases	[J] 20.3, [F] VI.30, [H] 4.2
02/21	How much trouble is infinity?	Zorn's lemma Infinite-dimensional vector spaces Maximal ideals and ranks	TBA
02/26	When can we pretend we're a vector space?	Direct sums Free quotients and summands Torsion-free modules	[DF] 10.3, [H] 4.5
02/26	How close are PID's to fields?	Modules over PID's	TBA